

# In search of the perfect fluid

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**Abstract.** Shear viscosity measures the amount of internal friction in a simple fluid. In kinetic theory shear viscosity is related to momentum transport by quasi-particles, and the uncertainty relation implies that the ratio of shear viscosity  $\eta$  to entropy density  $s$  is bounded by a constant multiplied by  $\hbar/k_B$ , where  $\hbar$  is Planck's constant and  $k_B$  is Boltzmann's constant. A specific bound has been proposed on the basis of string theory. In a large class of theories that can be studied using string theory methods the constant is  $1/(4\pi)$ . Experiments at RHIC indicate that  $\eta/s$  of the quark gluon plasma is close to this prediction. We will refer to a fluid that saturates the string theory bound as a perfect fluid. In this contribution we summarize the theoretical and experimental information on the fluidity of the main classes of strongly interacting quantum fluids.

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## INTRODUCTION

Experiments at the relativistic heavy ion collider indicate that the quark gluon plasma behaves more like a strongly coupled liquid than like a weakly coupled plasma. The most dramatic manifestation of this liquid-like behavior is the observation of almost ideal hydrodynamic flow in heavy ion collisions [1]. This experimental result is in contrast to earlier experiments at lower energies, and to theoretical expectations based on weak-coupling QCD.

Corrections to ideal hydrodynamics are governed by dissipative terms. The magnitude of these terms is determined by transport coefficients, in particular shear viscosity, bulk viscosity, and thermal conductivity. We will see that for very good fluids the main source of dissipation is shear viscosity. Shear viscosity can be defined in terms of the friction force  $F$  per unit area  $A$  created by a shear flow with transverse flow gradient  $\nabla_y v_x$ ,

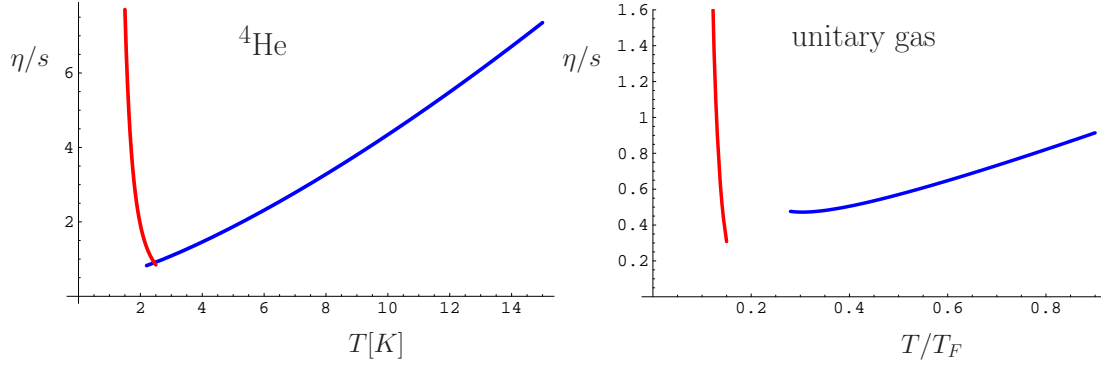
$$\frac{F}{A} = \eta \nabla_y v_x. \quad (1)$$

In a weakly coupled gas of quasi-particles the shear viscosity can be estimated as

$$\eta = \frac{1}{3} n p l_{mfp}, \quad (2)$$

where  $n$  is the density,  $p$  is the average momentum of the particles, and  $l_{mfp}$  is the mean free path. The mean free path can be written as  $l_{mfp} = 1/(n\sigma)$  where  $\sigma$  is a suitable transport cross section.

The shear viscosity of a good fluid is small. But just how small can  $\eta$  get? Danielewicz and Gyulassy pointed out that the Heisenberg uncertainty relation imposes a bound on the product of the average momentum and the mean free path,  $p l_{mfp} \geq \hbar$ , and concluded



**FIGURE 1.** Kinetic theory prediction for the ratio  $\eta/s$  in units of  $\hbar/k_B$  as a function of temperature in liquid  $^4\text{He}$  (left panel) and for a dilute Fermi gas at unitarity (right panel). The temperature of helium is given in Kelvin. For the dilute Fermi gas  $T$  is given in units of the Fermi temperature  $k_B T_F = (3\pi^2 n)^{2/3}/(2m)$  where  $m$  is the mass of the atoms and  $n$  is the density. The red curves show the low temperature expansion, and the blue curve is the high temperature expansion.

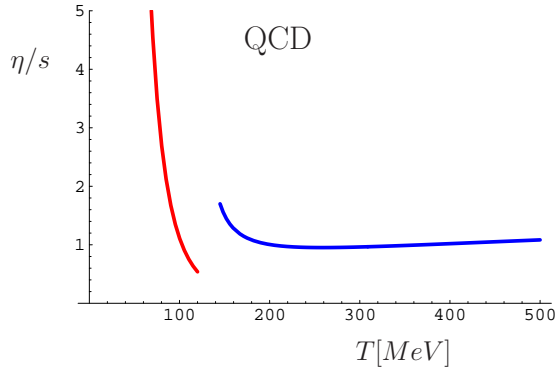
that  $\eta/n \geq \hbar$  [2]. For relativistic fluids it is more natural to normalize  $\eta$  to the entropy density rather than the particle density. Using  $s \sim k_B n$  we conclude that  $\eta/s \geq \hbar/k_B$ .

This is not a precise statement: The kinetic estimate in equ. (2) is not valid if the mean free path is on the order of the mean momentum. An important recent development is the discovery that one can compute the strong coupling limit of the ratio  $\eta/s$  in certain extensions of QCD using methods borrowed from string theory. Policastro et al. showed that in  $\mathcal{N} = 4$  supersymmetric QCD the strong coupling limit of  $\eta/s$  is equal to  $\hbar/(4\pi k_B)$  [3], and it was later conjectured that  $\eta/s \geq \hbar/(4\pi k_B)$  is a universal lower bound, valid for all fluids [4].

## PERFECT FLUIDITY

We will refer to a fluid that saturates the proposed bound as a “perfect fluid”. A perfect fluid is not only characterized by very small dissipation, but also by the fact that hydrodynamics has the largest possible domain of validity. In a typical fluid hydrodynamics is an effective description of low frequency, long wavelength fluctuations. In a perfect fluid hydrodynamics is valid down to scales as small as the inter-particle spacing.

Are there any perfect or nearly perfect fluids in nature? A perfect fluid has to be a quantum fluid (because  $\eta$  is on the order of  $\hbar n$ ), and it has to be strongly interacting (because in a weakly interacting system the mean free path is large). The main examples of strongly interacting quantum fluids in nature are i) liquid  $^4\text{He}$ , a strongly coupled Bose fluid, ii) dilute Fermi gases at unitarity (systems in which the scattering length was tuned to infinity using a Feshbach resonance), iii) strongly coupled plasmas, in particular the quark gluon plasma.



**FIGURE 2.** Kinetic theory prediction for  $\eta/s$  in QCD. The red curve shows the low temperature expansion, and the blue curve is the results of the high temperature expansion.

## TRANSPORT THEORY

In this section we summarize theoretical information about the transport properties of these quantum fluids. We will concentrate on results based on kinetic theory. For more details and for a discussion of other approaches (holography, linear response theory, etc) we refer the reader to our recent review [5].

Kinetic theory applies whenever the fluid can be described in terms of quasi-particles. For many fluids this is the case in both the high and the low temperature limit. Typically, at high temperature the quasi-particles are the “fundamental” degrees of freedom (atoms in the case of helium and the dilute Fermi gas, quarks and gluons in the case of QCD) whereas the low temperature degrees of freedom are composite (phonons and rotons or just phonons in the atomic systems, and pions in the case of QCD). In the regime in which kinetic theory applies the ratio  $\eta/s$  is always parametrically large. This leads to a characteristic “concave” temperature dependence of  $\eta/s$  [6]. Kinetic theory is useful in constraining the location of the viscosity minimum, but it cannot reliably predict the minimum value of  $\eta/s$ .

Kinetic theory results are summarized in Figures 1 and 2. We observe that all three fluids are expected to exhibit viscosity minima with  $(\eta/s)_{min} < \hbar/k_B$ . The minima occur in the vicinity of the phase transition,  $T \sim 2.2$  K in the case of helium,  $T \sim 0.15T_F$  for the dilute Fermi gas, and  $T \sim (150 - 180)$  MeV in QCD.

## EXPERIMENTAL SITUATION

Finally, we summarized the experimental situation for the the three quantum fluids discussed in this contribution.

- Liquid helium has been studied for many years and its shear viscosity is well determined. The minimum value of  $\eta/s$  is about  $0.8 \hbar/k_B$  and is attained near the endpoint of the liquid gas phase transition. The ratio  $\eta/n$  has a minimum closer to the lambda transition. Even though  $\eta/s < \hbar/k_B$  the transport properties of liquid helium can be quantitatively understood using kinetic theory [7].

- Strongly interacting cold atomic Fermi gases were first created in the laboratory in 1999. These systems are interesting because the interaction between the atoms can be controlled, and a large set of hydrodynamic flows (collective oscillations, elliptic flow, rotating systems) can be studied. Current experiments involve  $10^5 - 10^6$  atoms, and the range of temperatures and interaction strengths over which hydrodynamic behavior can be observed is not large. An analysis of the damping of collective oscillations gives  $\eta/s \sim 0.5$  [8, 9]. Even smaller values of  $\eta/s$  are indicated by recent data on the expansion of rotating clouds [10].
- The quark gluon plasma has been studied in heavy ion collisions at a number of facilities, AGS (Brookhaven), SPS (CERN), RHIC (Brookhaven). Almost ideal hydrodynamic behavior was observed for the first time in 200 GeV per nucleon (in the center of mass) Au on Au collisions at RHIC. These experiments are difficult to analyze - the initial state is very far from equilibrium and not completely understood, final state interactions are important, and the size and lifetime of the system are not very large. Important progress has nevertheless been made in extracting constraints on the transport properties of the quark gluon plasma. A conservative bound is  $\eta/s < 0.4$ , but the value of  $\eta/s$  that provides the best fit to the data is smaller,  $\eta/s \sim 0.1$  [11].

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